Team Control Number
13152
Problem Chosen
A
2022
HiMCM
Summary Sheet

Summary

Animals or insects like bees are all important animals to the agriculture. Honeybees can pollinate different plants while they are trying to find nectars in the flowers. Pollination done by honeybees can help the plants to grow further and produce the crops people need for living and develop economy. In recent years, people created a term called Colony Collapse Disorder (CCD) to describe the decline of honeybee populations around the world. Bee decline can cause a lot of problem which may influence the agricultural production and have bad effects on the world's economy. To prevent the significant loss caused by the bee decline, people have to keep an eye on the population of the bees and human intervention, if necessary, is also needed to reduce the decrease of the population of honeybees. Our group's work is to generate a model to find out the relationship between different kinds of natural factors with the population of honeybees in the world.

Our model considered about the environmental conditions, vegetation area, and food which we think can influence the population of honeybees. Because of the complex situation in different honeybee colonies, we did some simplification and try to find the relationships between different variables in order to get the correct output which symbolizes the population of the bees. The bees' main source of food is the flowers. They get nectars and pollens from the flowers and make them into honeys as their food. In different time in a year, the times honeybees go out to find food change since the climate changes, also the bees have to fly farther than before after some time because the population increases. Each bee has its own time to live, which means after some time, the amounts of bees will decrease naturally. We also considered some situations that bees die quicker than usual because of the harsh living conditions.

This model can help us to forecast the population of honeybees after a period of time which is comparatively accurate after considering about many conditions that may influence the population of the honeybees. Although the model isn't completely accurate, we still think that it can help people to predict the bee population in the future and make changes to the conditions which have problems on time to stop the loss from happening.

2022.11.15

tHe Need For Bee

Honeybee is a kind of animal which is important to agriculture since it help pollinate the plants in order to grow crops which is very important to we human beings in many ways. However, the population of honeybee is decreasing in the world because of some sort of problems. To prevent this from developing bad effects which may seriously damage human society, we have to find the relationships between many kinds of conditions with the population of honeybees. As we all know, the most important thing for living things is enough energy to support them keeping alive, so the area honeybees can find their food is the first thing we considered about. A flower can only be pollenated for several times, so with the increasing of the honeybee colonize, the area honeybee find food is going to be increased too. Animals also have their lifetime, which means though they have enough energy, they will still die when the time has come, so we also have to analyze this point. We all know that when honeybees are working, they use some of the energy they restored in their body, so that their lifetime will be reduced when they go out and work for one time. Also, the seasons is also important while thinking about this, because different animals have different living habits in different seasons. Because after summer, there aren't many flowers for the bees to pollinate, bees rarely go out during that time, so that bees can live for a longer time. But what if they didn't have enough food? We all know that bees eat honey for food, and they restored a lot of them during the first half of the year. Later if they eat all the honey, what will they do? They won't do nothing and die of starvation. They will eat the eggs and larva which they can find in their hive. After considering about the time used to make honey and many other unpredictable conditions, we have our model which can be used to solve the problem.

Contents

1. Introduction
1.1. Background
Since the honeybees pollinate about a third of all crops, they are of enormous ecological importance to the human existence. However, the plight of the essential bees is not encouraging. The global bee population has been declining for a long time. In 2007, the decline in bee populations gave rise to a new term, colony Collapse Disorder (CCD), whose causes and solutions are still being investigated. Factors including pesticides, environmental conditions, vegetation area, and so on, all affect bee reproduction and survival.

1.2. Problem Restatement
This problem requires us to design a model to forecast the population of a honeybee colony over time, based on the current situation of bee decline. We need to look for information about the factors that affect bee populations and design models based on those factors.

We need to conduct sensitivity analysis on our model to determine the factors that will have the greatest impact on honeybee populations. We need to make some changes to the constraints to see how they affect the result to get the importance of each factor.

Finally, suppose that we have a parcel of 20-acres land containing crops that need to be pollinated. We need to make a model to forecast how many beehives will be needed to support the needs of pollination in this area.

2. Assumptions and Justifications
2.1. General Assumptions
Assumption 1: There aren’t any colony fissions, and queen bee never grow old

Justification 1: Aging of the queen bee leads to the increase colony fission and a decrease in the number of eggs laid in the hive. But colony fission does not really affect the overall bee population.

Assumption 2: Until the newborn bees become adults, they cannot do any work and do not need any feeding (the food they need is stored in the spleen when they lay eggs).

Justification 2: Since the food needed by newborn bees is in the spleen when the bees lay eggs, and they can barely work as they grow, we assumed that they don't need food or work until they are adults.

Assumption 3: The spawning coefficient of bees in each season is constant and constant: spring: 1.5 Summer: 1. Autumn: 1.2 Winter: 0.8

Justification 3: Since the amount of nectar collected in different seasons is different, it will affect the laying of bees. We argue that temperature influences how well bees lay eggs. We showed the different effects of the four seasons on bee spawning in terms of the spawning coefficient.

2.2. Symbol Notation

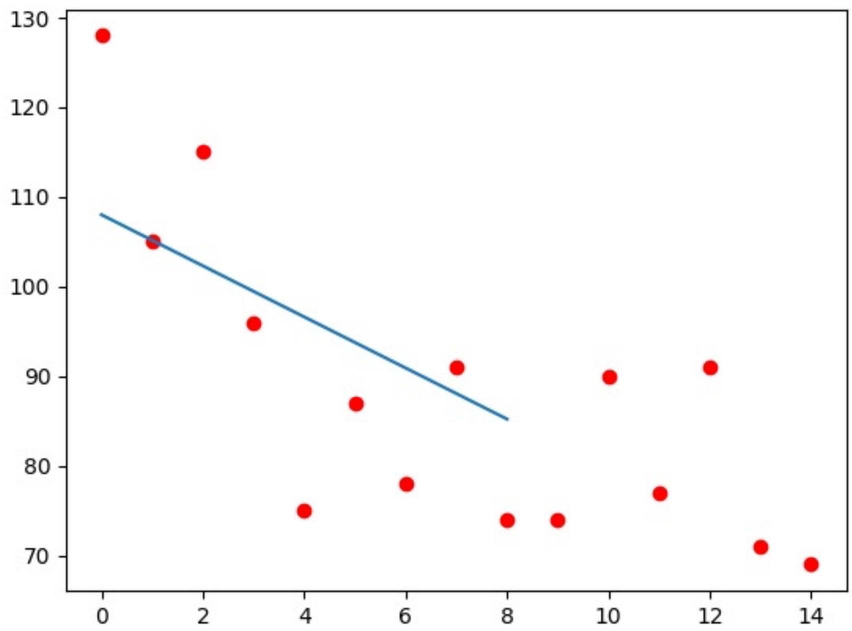
Symbol	*Description*
S_h	The area of land(20-acres), which is a constant
A_h	The number of flowers visited per bee per day=1000
C_{cmax}	The maximum number of times each flower can be pollinated by bees per day
A_b	The number of bees
$N_m g$	The honey a bee can produce every day
$A_{lyy}[i]$ $(1<=i<=20)$	Number of eggs, larvae and pupae
T_g	The time it takes to go from egg to adult
	The number of flowers visited per bee per day
C_g	The survival rate of mature bees=0.81
T_n	The time when pollen is turned into honey
A_m	The amount of honey consumed per egg
F_m	The amount of honey left over
M_p	The amount of pollen a bee gathers per visit to a flower=24mg
$M_d[i]$. $(1<=i<=8)$	Pollen that bees gather every day
D	The number of eggs the queen bee lays per day
B	Queen bee lays eggs per day B=800 eggs per day
C_T	Spawning coefficient (temperature & abundance of resources)
H	The amount of honey that bees produce each day
a	The length of the field’ s sides

Y	Number of hives
L	The distance that a single bee travels
W	The workload of each bees

3. The Population Model

3.1. Analysis of past bee populations

Average honey production per bee colony in North Dakota, USA from 1998 to 2012 (in lbs)[2]



We used the least square method to analyze past bee population data

The difference between the function and the known point set (least square meaning) is minimized by one-time fitting formula, which means that several discrete function values of a function are known and the undetermined coefficients in the function are adjusted once.

The honey production function $\boldsymbol{y = -2.85x + 108.01667}$ was fitted once for the average honey yield of each bee colony in North Dakota (the largest honey-producing state in the United States) from 1998 to 2012, and the goodness of fit R was obtained as **0.74145** (the fitting degree of regression line to the observed value).

This data tells us that there has been a downward trend in honey production in North Dakota in recent years, and honey production can tell us a lot about the state and overall population of bees.

3.2. Some information about bees

Before the analysis, we looked up some information about bees. These information helps us model the habits and conditions of bees.

Such as:

The development time of all bee eggs to adult was fixed at 20 days, and the survival rate was fixed at 0.81. Honey is brewed for a fixed period of eight days.

The rest of the bee information will be clarified in subsequent analyses

3.3. Modeling based on the life characteristics of honeybees

First, the flower field must be a square with a side length of a and an area of S_h, assuming a constant number of 40 flowers per square meter (artificially planted). We think that the influence area of each hive is a circle. When the pollen is sufficient, a bee will preferentially collect the flowers which is close to the nest, and he can pick 100 flowers per trip, which means that he visits 2.5 square meters of flower field (S_f). Each flower can be "visited" by 400 bees. Each bee regularly makes 15 trips a day. The nectar resets in the next day.

Therefore, we can approximately assume that bees will pick up all the flowers in a $\mathbf{S_f}$//400 square meter circle, that is, the area S covered by a bee can be expressed as

$$\boldsymbol{S} = \frac{\mathbf{2.5}}{\mathbf{400}} = \mathbf{0.00625}$$

So, the total area covered is:

$$\sum \boldsymbol{S} = \boldsymbol{S_0} + \boldsymbol{S}$$

Here we do not consider the flight time of the bee, only the distance it flies, L is the length of a round trip of the bee. And the total coverage will dynamically affect the size of L, because later bees need to travel farther to collect honey. The distance he needs to travel is the distance to the outermost area of the current area, which is twice the radius of $\sum$ S (because it is a round trip) plus the circumference of the circle. Although this would allow later bees to travel more distance, the radius increases more slowly as S increases, so we think it's reasonable.

$$\boldsymbol{L = 2\sqrt{\frac{S}{\pi}} + 2\sqrt{S\pi}}$$

The initial work load (W) of each bee is 1500, which is equivalent to the Hp of each character in the game. W naturally subtracted 10 a day due to the bees' basic metabolism. And his flight distance will also reduce W. The coefficient CL of L is different in different seasons, which is 1.5 in spring, 4 in summer, and 0 in autumn and winter. In order to simulate random factors in real situations, we have a random coefficient R (0.8~1.2).

$$\boldsymbol{\Delta W = \frac{L}{2} C_L R}$$

So, the bee has to subtract Δ **W** from **W** every time it goes back and. Forth.

$$\boldsymbol{W = W_0 - \Delta W - 0.666}$$

Bees would die if W were less than or equal to zero.

When W is less than or equal to zero, bees die. At the time of. Implementation, we counted the work load of each bee and ranked them in order from smallest to largest. We default that bees with less work load left will be assigned to the easier task of gathering nectar closer to home.

It's worth mentioning that we think the honey is made with the return of the bees. In each flower visited by bees, $\mathbf{M_p}$mg of honey can be collected, but this $\mathbf{M_p}$ will be affected by the season -- that is, the temperature. So our honey yield will be affected by the climate coefficient $\mathbf{C_T}$, which we believe is consistent with the work load. So the amount of pollen ($\mathbf{M_d}$) that a bee colony of size Ab can collect per day is

$$\mathbf{M_d = 150 A_b M_p C_T}$$

Since honey takes eight days to ferment, the pollen count is stored in a queue H[8] of length 8 (first in, first out). The number of pollen counts in H is pushed back by one each day, and the index of each quantity is its fermentation days. We believe that the mass ratio of honey and pollen is 0.25:1, which is converted (because 80% of water is removed in the process of honey fermentation), that is, the newly added H[1] is:

$$H[1] = \frac{1}{4}M^d$$

The amount of Fm added to the honey reserve every day is H[8]

Before we go on to discuss honey reserves, we need to know how many eggs the queen lays each day, because each egg is produced at the cost of a certain amount of honey (**A_m**). The number of eggs laid per day by a queen bee is composed of two numbers -- the queen spawning base of 800 eggs per day, and the climatic factor **C_c**, which is 1.5 in spring, 2.5 in summer, 1 in autumn, and 0 in winter, since the queen bee stops laying eggs when the temperature drops to 5° C. Like honey, there is also a queue for egg production, and the queue length is 20 (3 days for egg incubation, 5 days for larva pupae, and 12 days for pupae metamorphosis into adult). Therefore, **A_{lly}**'s new data every day is **A_{lly}[1]**

$$A_{lly}[1] = 800C_c$$

The change in the colony is multiplied by the survival rate of the young：

$$A_b = A_{b0} + 0.81A_{lly}[20]$$

Now, returning to the issue of honey storage, the amount of honey consumed by the queen bee per egg is Am=120mg. Each bee consumes 60mg of honey a day

So the formula for honey storage is：

$$F_m = F_{m0} - 120A_{lly}[1] - 60A_b + H[8]$$

But what if Fm goes negative?

The bees will start eating their own eggs

A juvenile bee is thought to weigh 1000mg and the total ratio of sugar to protein is 0.1

$$IF_m = 100 \times 0.1A = 100A$$

The A will be subtracted from the **A_{lly}** and eaten preferentially from the egg in the front

Bees will "consume" their pups until IFm can turn Fm back positive, or all pups are consumed, i.e., the A range is:

$$\boldsymbol{A} \leq \sum_{i=1}^{20} \boldsymbol{A}_{lly}[\boldsymbol{i}]$$

If the number of young is not enough to eat, that is, when **F_m+IF_m<0**, some worker bees will starve to death. We assume that E bees will starve to death if bees do not eat for one day.

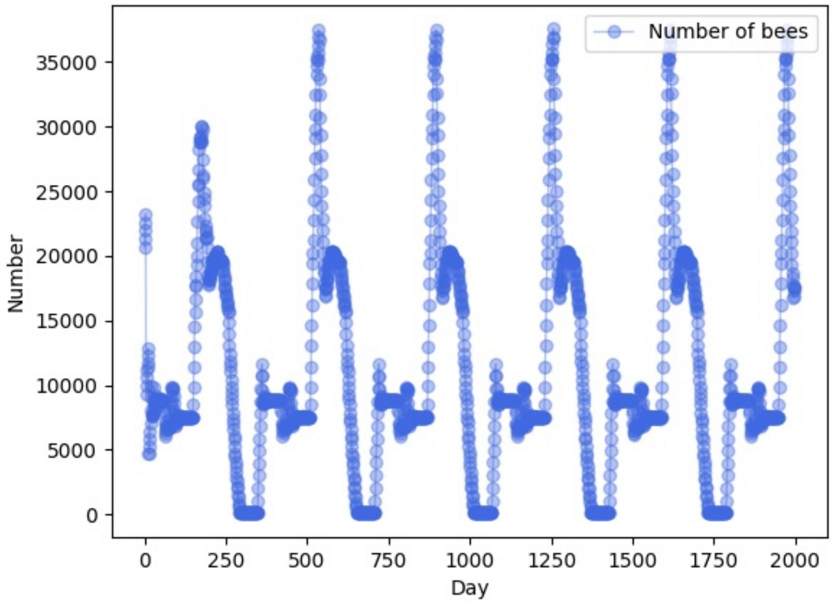
$$\boldsymbol{E} = \left\lceil \frac{|\boldsymbol{F}_m|}{\Delta \boldsymbol{H}} \right\rceil$$

If No bee is going to starve to death , E is zero.
It's worth adding that when bees ran low on food, they prioritized getting rid of older, less productive bees after eating the larvae. But in our model, individual changes in bee productivity are ignored, so of course, this is the part of the colony reduction that we think is negligible.
Therefore, the updated colony number formula is:

$$\boldsymbol{A}_b = \boldsymbol{A}_{b0} + \mathbf{0.81}\boldsymbol{A}_{lly}[\mathbf{20}] - \boldsymbol{E}$$

3.4. The result

We simulated the model with a program and found the normal bee population change over 2000 days.



We found that the number of bees would change from season to season in the same way as described above, under conditions that did not differ greatly from year to year.

We input an initial number of 20000 bees in the model. The model will run for 2000 days from April 1. The initial honey inventory is 10kg. Taking one year as a cycle, after the rapid growth of the number of bees in summer, there was a relatively small but rapid decline in autumn, and soon ushered in a big decline in winter, with the lowest number of bees approaching 150.

4. Sensitivity Analysis

Graph 1:

It shows the relationship between the initial bee population and the trend of bee population change.

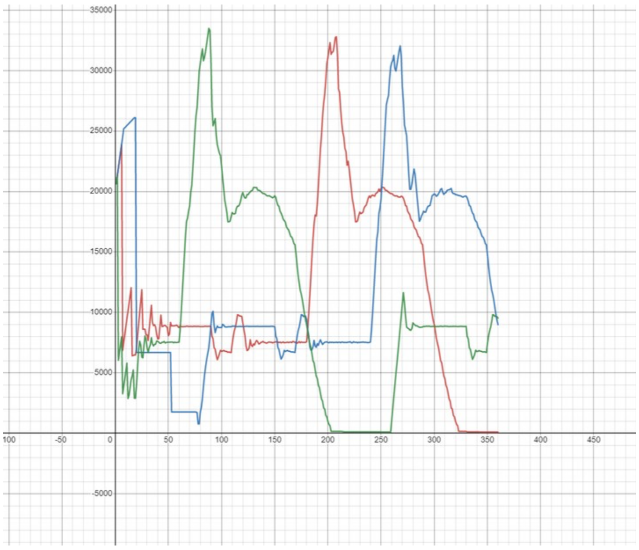
The overpopulation can cause many bees to die from overwork. The workload of the model restrict the number of the bees' population as consumption of the honey storage have the same effect, so that high initial bee numbers are led to similar trends.

Graph 2:

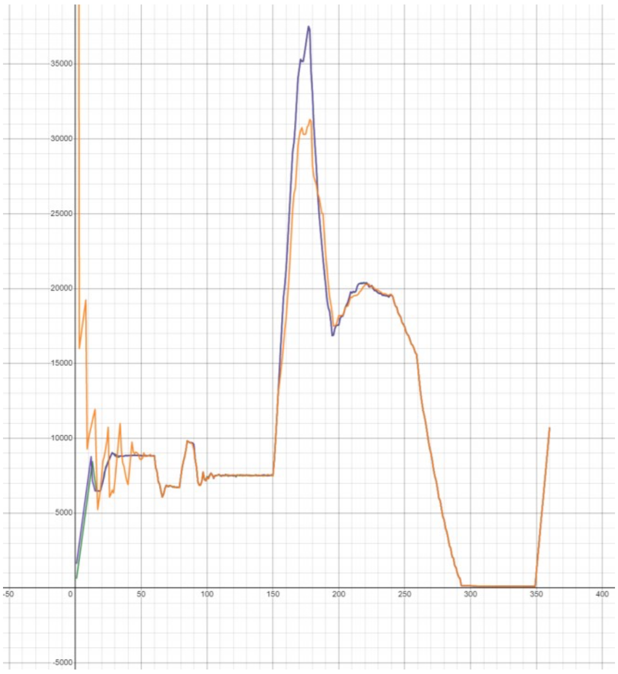
It shows the relationship between honeybee initial month and bee population change.

As we can see, we chose January (blue), March (red), and July (green) as our data points. We can see that changes in the initial month only delay or advance the trend that leads to changes in bee populations. The overall trend is similar regardless of the initial time point.

Graph 1:



Graph 2:



5. Prediction of Honeybee hive demand

In the code that we have, a function maxn record the maximum number of $\sum S$. But as the number of the bees is very unstable, they can't always fulfill this maxn. So we consider the outer part of the space is similar to electron cloud, which means it will have intersection with other hive's coverage area, which is a leaf shape area.

So the equation for this area is :

$$S_i=\frac{\varphi}{180}\pi r^2 - r^2 * sin\varphi$$

In the equation φ=45°,and maxn is 3500m^2 ,which means r=33.378m S_i=87.223m^2

To cover a 81,000m^2 parcel of land need

$$\frac{81{,}000-174.446}{3500-174.446}=24$$

So we need 24 honeybee hives to support pollination of a 20-acre (81,000 square meters) parcel of land containing crops that benefit from pollination.

6. Evaluation

6.1. Strengths

Our model considers several factors related to bee behavior, defines a number of relevant variables, and simulates a more realistic state of bee population change. We use intuitive programs to show bee populations at different points in time under the influence of a series of Settings related to temperature, bee behavior, and environmental resources.

In our program, we can get the daily variation of the number of bees and the final number of bees by simply entering the starting number of bees and the full bee month.

6.2. Weaknesses

Most of our models assume that the external factors for bees are pretty much the same every year. So the bee population always returns to a similar size. So far, we've only analyzed how bee populations are affected by external factors. But it's important to note that from one year to the next, the environment in which bees live can change dramatically. But global temperatures are influenced by many factors that may be far greater than previously thought. Therefore, to some extent, our group's model only established a general situation of bee population change over time but could not better simulate unexpected events with low probability, such as sudden rise of global temperature, large-scale sudden natural disasters and so on.

7. Conclusion

In our model of bee populations, we have a long-term approximate trend of bee population changes. In our analysis, we have considered very carefully the habits of bees and the influences of external factors, so we conclude that the bee population will always reach a similar value after a period of change.

But why is the annual bee population apparently declining?

In our model, we generally assume that the external factors are approximate from year to year, because it is difficult to predict future changes in the factors affecting bees. These factors could change under the influence of global warming, or bee populations could suddenly decline under the influence of a global virus. And that's why we think it's these external factors that are causing bee populations to decline year after year. They affect the time of year when bee populations tend to decline, causing bee populations to decline even more than usual.

Reference

[1]Honey bee pollen data
https://www.kaggle.com/datasets/ivanfel/honey-bee-pollen
[2]Honey production in USA
https://www.kaggle.com/datasets/jessicali9530/honey-production
[3] A model of honeybee intracolonial population dynamics and resource management
by Thomas Schmickl*, Karl Crailsheim

Appendix

[1]
```
import matplotlib.pyplot as plt
import math
def linefit(x , y):
   N = float(len(x))
   xf1,yf1,xf2,yf2,sxy=0,0,0,0,0
   for i in range(0,int(N)):
      xf1  += x[i]
      yf1  += y[i]
      xf2 += x[i]*x[i]
      yf2 += y[i]*y[i]
      sxy += x[i]*y[i]
   a = (yf1*xf1/N -sxy)/( xf1*xf1/N -xf2)
   b = (yf1 - a*xf1)/N
   r = abs(yf1*xf1/N-sxy)/math.sqrt((xf2-xf1*xf1/N)*(yf2-yf1*yf1/N))
   return a,b,r
x=[ 0,1,2,3,4,5,6,7,8,9,10,11,12,13,14]
```

```
y=[128,105,115,96,75,87,78,91,74,74,90,77,91,71,69]
a,b,r=linefit(x,y)
print(" y = %10.5f x + %10.5f , r=%10.5f" % (a,b,r) )
plt.plot(x,y,'ro')
plt.plot([0,8],[b,a*8+b])
plt.show()
```

[2]
```
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
#%matplotlib inline
import warnings
import math

num=[0 for i in range(2000)]
i=0
filename='C:/Data/data.txt'
with open(filename) as data:
    for line in data:
        num[i]=int(line)
        #print(line)
        i+=1


x=[0 for i in range(2000)]
for i in range(2000):
    x[i]=i
#print(num)

plt.plot(x, num, 'ro-', color='#4169E1', alpha=0.4, linewidth=1, label='Number of bees')

plt.legend(loc="upper right")
plt.xlabel('Day')
plt.ylabel('Number')
plt.show()
```

[3]
```
#include <bits/stdc++.h>
using namespace std;

int n = 20000;
double W[120000];

double Ql(double s) {
        double L = sqrt(s / 3.14) * 2 + 2 * sqrt(s * 3.14);
        return L;
}

void death(double num, int i) {
        if (num <= 0) {
                W[i] = 1500;
                n--;
        }
}

int main() {
```

```
	freopen("shuju.out", "w", stdout);
	int d = 0, day;
	double s = 0;
	cin >> n >> day;
	double maxn = 0;
	for (int i = 0; i < 120000; i++) {
		W[i] = 1500;
	}

	//num[120000]=1;
	sort(W, W + n);
	int date = 0, month = 0;
	double Cl, Cc;
	double Fm = 0, H[9] = {0, 0, 0, 0, 0, 0, 0, 0};
	cin >> month >> date;
	cin >> Fm;

	int ally[22] = {800, 800, 800, 800, 800, 800, 800, 800, 800, 800, 800, 800, 800, 800, 800, 800, 800,
800, 800, 800, 800};
	int e, A;

	for (d = 0; d < day; d++) {
		s = 0;
		e = 0;
		if (Fm < 0) {
			double IF = 0;
			IF = abs(Fm);
			A = IF / 100;
			for (int q = 0; q < 20; q++) {
				if (ally[q] >= A) {
					ally[q] -= A;
					Fm += A * 100;
				} else {
					Fm += ally[q] * 100;
					A -= ally[q];
					ally[q] = 0;
				}
			}
		}
		if (Fm < 0) {
			double IF = abs(Fm);
			e = IF / 60;
		}
		if (date == 31) {
			date = 1;
			month++;
			//cout << n<<"\n";
			if (month == 13) {
				month = 1;
			}
		}
		if (month == 12 || month < 3) {
			Cl = 0.3;
			Cc = 0;
```

```
        }
        if (month >= 9 && month < 12) {
            Cl = 0.2;
            Cc = 1;
        }
        if (month >= 3 && month < 4) {
            Cc = 1.5;
            Cl = 1.1;
        }
        if (month >= 4 && month < 6) {
            Cc = 1.5;
            Cl = 1.25;
        }
        if (month >= 6 && month < 7) {
            Cl = 2;
            Cc = 2.5;
        }

        if (month >= 7 && month < 9) {
            Cl = 3;
            Cc = 2.5;
        }
        ally[0] = 800 * Cc;

        for (int k = 0; k < 15; k++) {
            int zan = n;
            for (int i = 0; i < zan; i++) {
                s += 2.5 / 400;
                double L = Ql(s);
                int rnum = rand() % 5;
                double R = 1 + (2 - rnum) / 10.0;
                double dD = L / 10 * R * Cl + 0.666;
                W[i] -= dD;
                //cout << num[i] << "  ";
                death(W[i], i);
                H[0] += 24 * Cl * 0.25;
            }
            sort(W, W + n);
        }
        maxn = max(maxn, s);
        for (int g = 7; g >= 0; g--) {
            H[g + 1] = H[g];
        }
        Fm = Fm + H[8] - n * 60 - ally[0] * 120;
        for (int p = 19; p >= 0; p--) {
            ally[p + 1] = ally[p];
        }
        n = n + ally[20] * 0.81 - e;
        date++;
        cout << n << "\n";
    }
    //cout << maxn;
    return (0);
}
```