Studies on Bee Colony Numbers and Pollination Efficiency

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Great thanks to Sunday Li and Maggie Wang for their contributions to this paper!

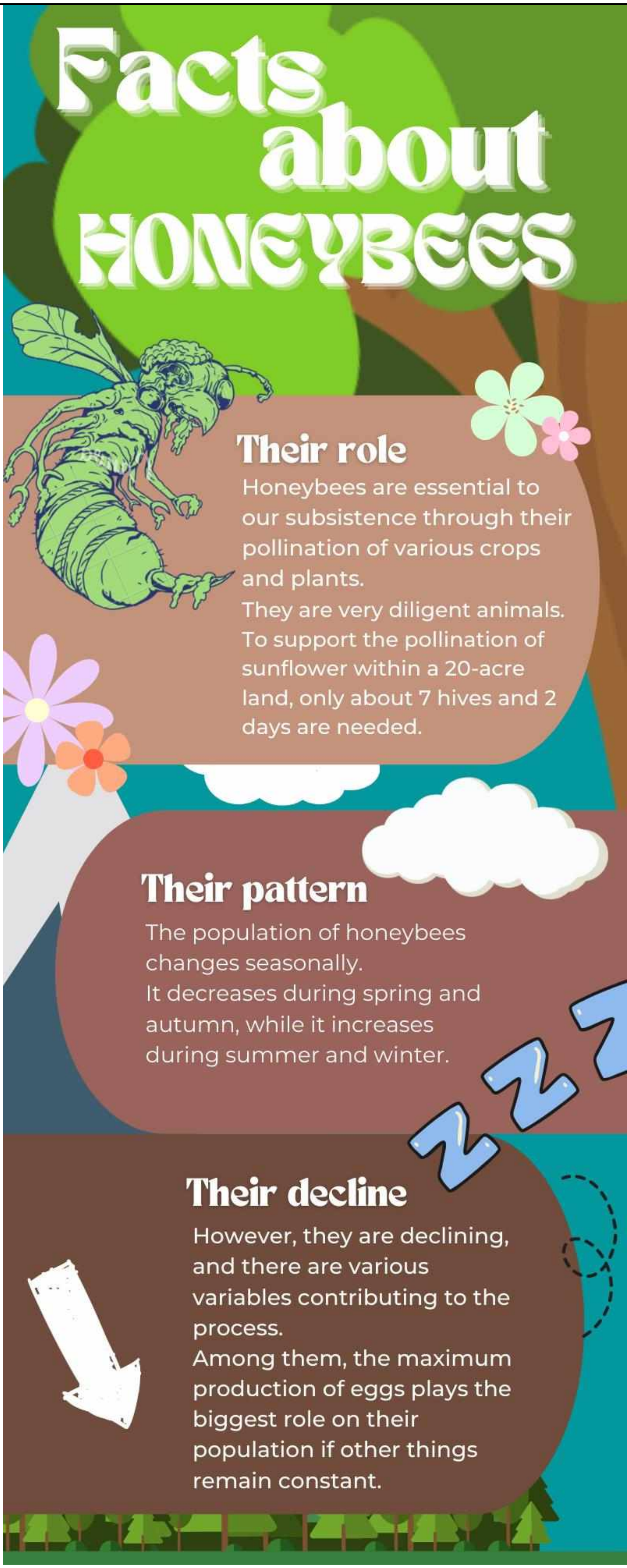
Bees, industrious animals we see in our daily life, are essential to the survival of human beings on earth for their vital role of pollinating many trees and plants. However, in 2007, the term Colony Collapse Disorder (CCD) was coined, indicating global declining of honeybee population. This decline in bee populations can be attributed to numerous factors. To yield the most effective solution, how honeybee population changes over time and what is the main factor contributing to the decline is important.

Correspond with the needs, we modeled the honeybee population overtime and used sensitivity analysis to find the main reason for the decline. To do that, we established certain assumptions and differential equations and functions based on the assumptions. After calculating using odeint() in python scipy, output graph of the model is obtained and it was determined that honeybee population fluctuates seasonally along with their food supply and the temperature in their environment. More specifically, food supply increases during spring while decreases in other three; the number of eggs, larvae, pupae, hive bees, forager bees, and all bees all decreases during spring and autumn while increases during summer and winter. Then, we changed each default variables- death rates of eggs, larvae, pupae, hive bees, forager bees, and the maximum production of eggs- to determine which one of them caused the largest fluctuation in honeybee population. It was found that maximum production of eggs is the main factor contributing to the phenomenon.

Even though the main factor has been found, solutions correspond with the questions need time for humans to develop, experiment, and apply. This time gap between the present phase and the moment when a safe and effective solution can be applied can still cause great loss in honeybee population, and thus great damage to human's agriculture and subsistence. As a result, we need to maximize the utilization of honeybees in order to reduce the potential risk.

To mediate the damage during the gap, we developed a model that shows how long and how many honeybee hives are needed to pollinate crops in a 20-acre land. To start with, we used the idea of cellular automata(CA), of which complex patterns are often observed from simple rules or limitations. Similarly, we established certain assumptions about honeybees and their hives and then the model. To solve it, we used matplotlib of python and yielded the wanted results with visualized output.

Keywords: Honeybee, Colony Collapse Disorder(CCD), differential equations, Cellular Automata(CA).


Facts about HONEYBEES
Their role
Honeybees are essential to our subsistence through their pollination of various crops and plants.
They are very diligent animals.
To support the pollination of sunflower within a 20-acre land, only about 7 hives and 2 days are needed.
Their pattern
The population of honeybees changes seasonally.
It decreases during spring and autumn, while it increases during summer and winter.
Their decline
However, they are declining, and there are various variables contributing to the process.
Among them, the maximum production of eggs plays the biggest role on their population if other things remain constant.

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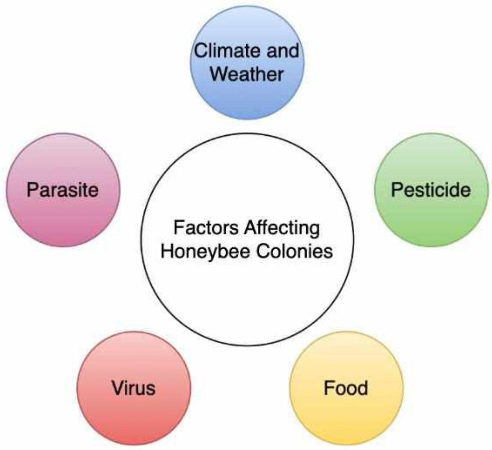
1 Introduction

1.1 Background

Bees, industrious animals we see in our daily life, are essential to the survival of human beings on earth. In addition to its well-known ability to produce honey, these insects provide the vital role of pollination of many trees and plants that provide food for our survival, so bees are closely linked to our lives. In 2007, the term Colony Collapse Disorder (CCD) was coined to describe the decline of bee populations worldwide. This indicates that the survival of bees is under threat. The decline in bee populations can be attributed to factors including parasites, virus, pesticides, environmental conditions- climate and weather, and available food supply. The decline in bee colonies will inevitably affect the yield of our various crops, and will also affect our daily lives. To prevent the decline, impacts of various variables and the one that contributes the most should be found for us humans to bring out the most effective solutions. Other than that, an estimation of the number of bee hives needed for a limited span of land should also be modeled for the intensification for the function of the honeybees, in other words, bring out their largest efficiency so to save the crops when their population is plummeting around the globe.

Correspond with the needs, we established models that can estimate the number of bees in the colony based on various factors, and find the factors that is the most influential to the population of bee colonies, which can provide constructive guidance for solving CCD problems in the future. In addition, our second model can also tell beekeepers how to reasonably arrange the number of beehives to complete pollination in an area through modeling, which not only greatly improves pollination efficiency, but also ensures the safety of bee colonies, which is of great significance.

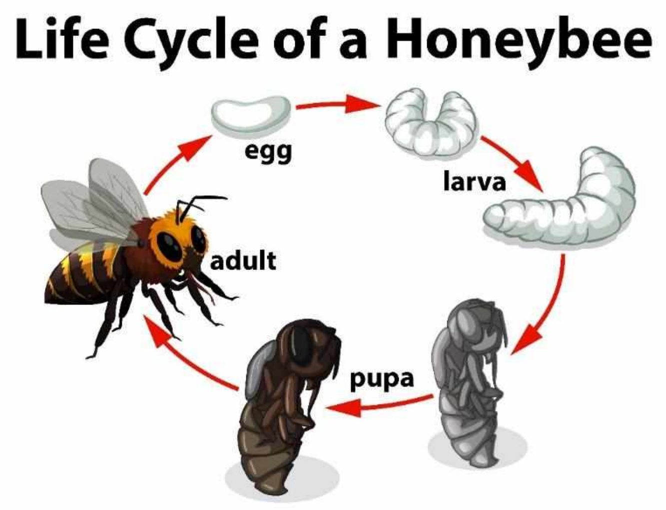


Credit: factors that affects honeybee colonies

1.2 Info. about honeybees

There are three castes of honeybees- workers, queens, and drones. They play different roles in a hive and live uneven amount of time. However, they have similar stages of development. In the first stage, an egg is produced by the queen and stays for three days. During the time period, it will be fed royal jelly if it is destined to be the next queen. Then, in the second stage, the egg turns into a larva and is fed continuously.

Going toward the third stage, the larva caps itself into a pupa. Once fully developed, the new adult bee breaks through the capping and goes to work its part. According to their castes, they stay different amount of time in each period.



Credit: https://www.vecteezy.com/vector-art/296715-life-cycle-of-a-honeybee

After adulthood, each caste of honeybee plays their own roles. Queens produce more bees and control the others through releasing pheromones. Drones increase the temperature within the hive through vibration, reduce the honey in storage through their huge amount of appetite, and mate with the queens. Workers do all the chores – collect resources, modify the temperature of the hives, feed the queens and the young, remove the dead, and protect the hives.

1.3 Problem Restatement

Question 1: We need to develop a model to determine the population of a honeybee colony over time. To do that, we first need to understand the basic information about honeybees, such as their habits, social division of labor and their life cycle. After that, we need to find out the numerous factors that affect the population of honeybees, for example their lifespans, egg laying rates and fertilized/unfertilized egg ratios. Then we need to analyze and model each factor, and finally combine the models together to make the population model of honeybees.

Question 2: We need to conduct sensitivity analysis on our model to determine which factors have the greatest impact on honeybee colony size. In order to do that, we need to change one factor at a time based on the results of the population model and find out the one that causes the biggest shift in honeybee population.

Question 3: We need to model and predict how many honeybees hives we will need to support pollination of a 20-acre (81,000 square meters) parcel of land containing crops that benefit from pollination. To achieve this purpose, we first need to understand the habitat of bees and establish relevant assumptions for our model. After that, we make our model based on the assumptions, and finally get the results by solving it.

Question 4: We need to create a non-technical infographic that presents our core results mentioned in this report.

2 The Population Model

2.1 Assumptions and Justifications

Assumption 1: Temperature and available food supply changes seasonally.

Justification 1: It was rooted from the sources we referred.

Assumption 2: The adequate temperature for all honeybees is 21℃

Justification 2: Same as justification 1.

Assumption 3: The total population involves honeybees in every stage of development.

Justification 3: Same as above.

Assumption 4: All honeybees considered in this report are in Shanghai, China.

Justification 4: Different honeybees have different behaviors and experience different environment, to make it easier, we have chosen our location- Shanghai.

2.2 Variable Chart

The table below shows all the variables we used in Problem 1 & 2:

Variable	Description
t	Time (day)
E	Number of eggs
L	Number of larvae
P	Number of pupae
H	Number of hive bees
F	Number of forager bees
N	Number of bees
f	Mass of bees' food (mg)
T	Temperature ($^\circ C$)

Table 1. Variable Chart for Problem 1

Function Notation	Description
$T(t)$	Temperature
$\varphi(t)$	Relative concentration of bees' food
$E_l(N, f, T)$	Egg laying rate
$S_E(H, E)$	Survival rate of eggs
$S_L(H, L, f)$	Survival rate of larvae
$S_P(H, P)$	Survival rate of pupae

$S_H(f,T)$	Survival rate of hive bees
$S_F(f,T)$	Survival rate of forager bees
$R(H,F,T,f)$	Recruitment rate between hive bees and forager bees

Table 2. Function Notation Description for Problem

Additional information about the parameters and default variables can be found in Table 3 and 4 in the Appendix.

2.3 Overview of the model

To model the population of honeybee, we have considered bees in different stages of development and adult bees in different roles. Other than that, we also took in consideration of the available food supply, temperature, and death rates caused by unspecified reasons. Since we are finding relations between numbers and time, we have established differential equations to model and employed method odeint() of python scipy to yield the results.

2.4 Algorithms

(For the parameters mentioned in some of the equations, we have attached them and their values considered in this report in the appendix.)

First, we start with the most fundamental equation. We assume that the total number of the bee colony is the sum of each life stage of bees, including eggs, larvae, pupae, hive bees and forager bees.

$$N = E + L + P + H + F \quad (1)$$

The differential equation suggests Assuming the daily rate of change of eggs is related to real-time egg laying rate, the transition from eggs to larvae and the survival rate of eggs, we establish the following differential equation:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = E_l(N,f,T) - \lambda_E \cdot E - E \cdot (1 - S_E(H,E)) \quad (2)$$

Similar to equation (2), we establish the rate of change of other variables in Equation (3), (4), (5) and (6). In equation (5) and (6), in order to simplify the problem, we assume that only hive bees will convert into forager bees, but forager bees will not convert into hive bees. In addition, they are all related to their own survival rate.

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \lambda_E \cdot E - \lambda_L \cdot L - L(1 - S_L(H,L,f)) \quad (3)$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda_L \cdot L - \lambda_P \cdot P - P(1 - S_P(H,P)) \quad (4)$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \lambda_P \cdot P - H \cdot R(f,T) - H(1 - S_H(f,T)) \quad (5)$$

$$\frac{\mathrm{d}F}{\mathrm{d}t} = H \cdot R(f,T) - F(1 - S_F(f,T)) \quad (6)$$

Then for the mass of food, an indispensable part of honeybee colony, it is split into the food gathered by forager bees and the food consumed by larvae, hive bees and forager bees since we assumed that only they can ingest the food. We add bell function 5 to display that they would take in less food during winter or in extreme hot weather.

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \varphi(t) \cdot f_g \cdot F - e^{-C_5 \cdot (T - T_{b_5})^2} \cdot (f_{c_L} \cdot L + f_{c_H} \cdot H + f_{c_F} \cdot F) \quad (7)$$

The following functions depict the detailed change in the variables.

We hypothesize that temperature is a sinusoidal function varied with time throughout a year. When time is near summer solstice, temperature reaches its maximum. When time is near winter solstice, temperature reaches its minimum. The temperature near spring or autumn equinox is assumed to be the arithmetic mean value of the maximum and the minimum temperature. Plus, we start from spring equinox in this report.

$$T(t) = \frac{T_{summer} + T_{winter}}{2} + \frac{T_{summer} - T_{winter}}{2} \sin(\frac{2\pi}{365}(t + t_{start})) \quad (8)$$

The form of function (9) is similar to function (8). It's also a sinusoidal function varied with time, which represents the relative food availability through a year. We mark the average value as 1, the minimum value as 0.25, and the maximum value as 1.75. Moreover, we assume that there's a time gap between the time of seasons and the time of florescence.

$$\varphi(t) = \frac{\varphi_{max} + \varphi_{min}}{2} + \frac{\varphi_{max} - \varphi_{min}}{2} \sin(\frac{2\pi}{365}(t + t_{start} + t_{flower})) \quad (9)$$

From equation (10) to (15), there's a fundamental rate for each function, which guarantees that the population wouldn't plummet to 0 very quickly. Plus, the survival rate of each life stage of bees is strictly controlled in the interval of $[0, 1]$.

We set an estimated value of maximum egg laying rate. In the actual egg laying rate, it's connected with the total number of bees, the food available in the hive and the temperature. When the total number of bees grows, queen bee would lay more eggs due to agglomeration effect. When the food in the hive is sufficient, there would also be more eggs laid by queen bee. However, when it comes to winter and the temperature drops, the egg laying rate will also drop.

$$E_l(N,f,T) = E_{l_{max}} \cdot (\sigma_{E_l} + (1 - \sigma_{E_l}) \cdot \frac{N}{\alpha_N + N} \cdot \frac{f}{\beta + f}) \cdot e^{-C_1 \cdot (T - T_{b_1})^2} \quad (10)$$

In function (11) - (13), we add a self-regulation factor into the survival rate. If the number of eggs, larvae or pupae is too large, then their survival rate will decrease because of limited resources and space. They would also be negatively affected by parasites, viruses and pesticides. To simplify the model, we set a coefficient that represents the death caused by these factors in the survival rate function.

$$S_E(H,E) = (\sigma_E + (1 - \sigma_E) \cdot \frac{H}{\alpha_H + H + \gamma_E \cdot E}) \cdot (1 - \mu_E) \quad (11)$$

$$S_L(H,L,f) = (\sigma_L + (1 - \sigma_L) \cdot \frac{H}{\alpha_H + H + \gamma_L \cdot L} \cdot \frac{f}{\beta_L + f}) \cdot (1 - \mu_L) \quad (12)$$

$$S_P(H,P) = (\sigma_P + (1-\sigma_P)\cdot \frac{H}{\alpha_H + H + \gamma_P \cdot P})\cdot(1-\mu_P) \tag{13}$$

Because only larvae, hive bees and forager bees eat, we consider that there is a connection between their survival rate and the food in the hive. A natural mortality rate is assumed to influence the survival of hive bees and forager bees. What's more, temperature would have different kinds of influence in the survival rate of hive bees and forager bees. Hive bees will have greater chance to survive the winter while hive bees cannot. Hotter weather will have greater influence on hive bees since they have to maintain high level of activity to support the whole hive.

$$S_H(f,T) = (\sigma_H + (1-\sigma_H)\cdot \frac{f}{\beta_H + f})\cdot e^{-C_2\cdot(T-T_{b_2})^2}\cdot(1-\mu_H)\cdot(1-m_H) \tag{14}$$

$$S_F(f,T) = (\sigma_F + (1-\sigma_F)\cdot \frac{f}{\beta_F + f})\cdot e^{-C_3\cdot(T-T_{b_3})^2}\cdot(1-\mu_F)\cdot(1-m_F) \tag{15}$$

The recruitment rate consists of basic recruitment rate and the rate varied with food availability. When the hive lacks food, there would be more hive bees turning into forager bees to sustain food supply. In winter, hive bees would stay inside the hive, so much less hive bees will go into the transition.

$$R(f,T) = (\delta + \delta\cdot(1-\frac{f}{\beta_R + f}))\cdot e^{-C_4\cdot(T-T_{b_4})^2} \tag{16}$$

2.5 Results

To solve the differential equations for results, we used the method odeint() in python scipy and yielded the following graph.

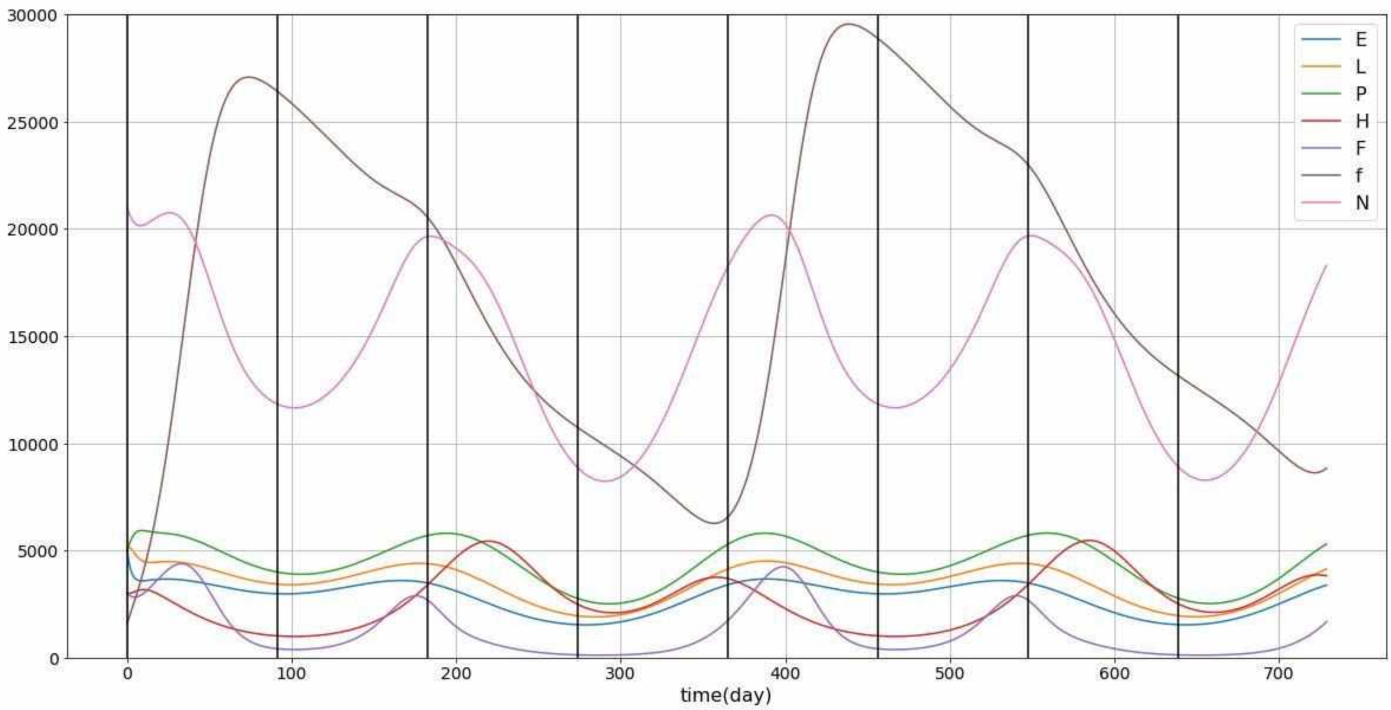


Figure 1. Bee population varied with time

The x-axis denotes the number of days passed after day 0. Each block separated by the vertical straight lines represent a season starting from spring. For example, the block from 0-100 represent spring, and the next block represent summer. The y-axis simply shows the number of whether egg, larva, pupa, hive bees, forager bees, food supply, or the total honeybee population.

In general, the graph shows the relationship of egg, larva, pupa, hive bees, forager bees, food supply, and the total honeybee population with time. All of the numbers shift seasonally as shown – food supply increases during spring while decreases in other three; the number of eggs, larvae, pupae, hive bees, forager bees, and all bees all decreases during spring and autumn while increases during summer and winter.

3 Sensitivity Analysis

In this section, the Population is tested by adjusting its inputs. The changes in result shows how changes in certain variables can influence the population of honeybees. We have made in total of 6 adjustments, the death rates (due to parasites, viruses and pesticides) of eggs(μ_E), larvae(μ_L), pupae(μ_P), the natural death rate of hive bees(m_H), and forager bees(m_F), and the maximum production of eggs($E_{l_{max}}$). After graphing the changes in population under the influence of each variable, we can find out the factor that contributes the most.

To calculate the fluctuation caused by each variable, we calculate the slope of each midpoints of the graphs, multiple it by the x value of the midpoint, and divide it by the y value of the point. The result value we will denote it as variable FL in the following sections. The larger FL is, the larger the caused fluctuation in the number of the total honeybee population.

3.1 Changing the death rate of larvae

The graph shows that as we increases μ_L from 0.00 to 0.10, N decreases from its original number to below 13000.

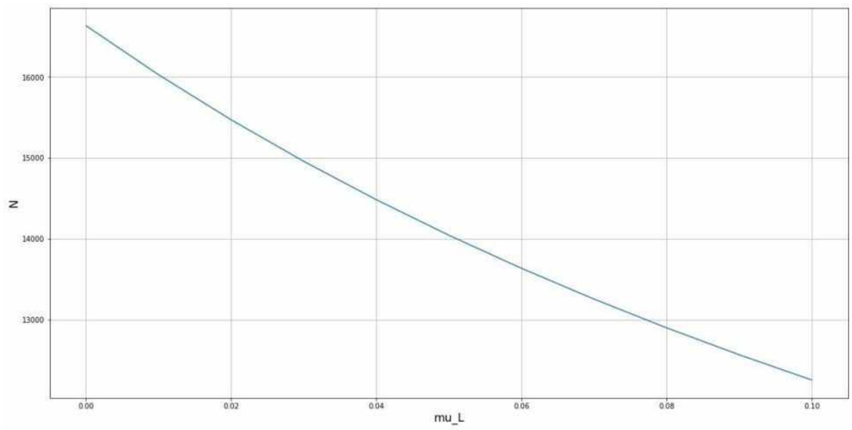


Figure 2. Changing the death rate of larvae

Sensitivity = 0.12

3.2 Changing the death rate of eggs

Similar to the graph above, the graph here shows that as we increases μ_E from 0.00 to 0,10, N decreases from its original number to below 13000.

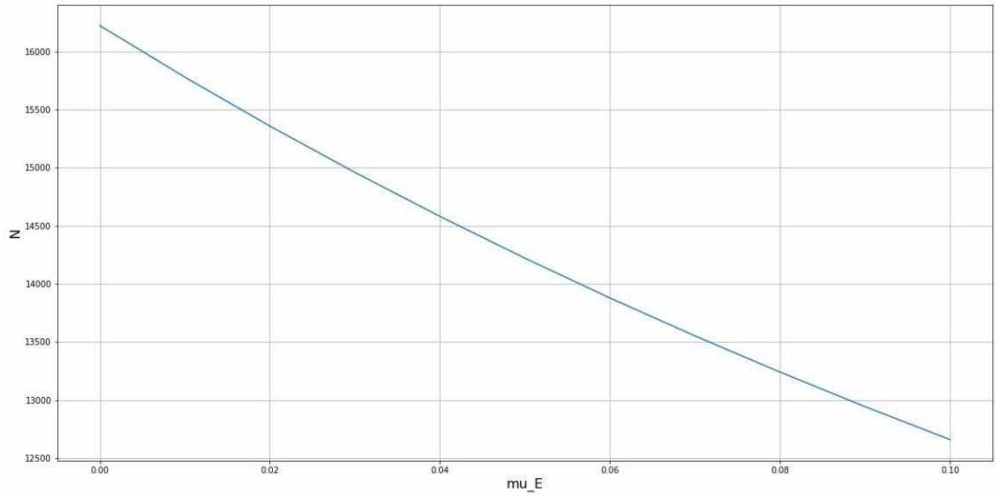


Figure 3. Changing the death rate of eggs

Sensitivity = 0.01

3.3 Changing the death rate of hive bees

Similar to the graph above, the graph here shows that as we increases m_H from 0.00 to 0,10, N decreases from bit larger than 15500 to nearly 12000.

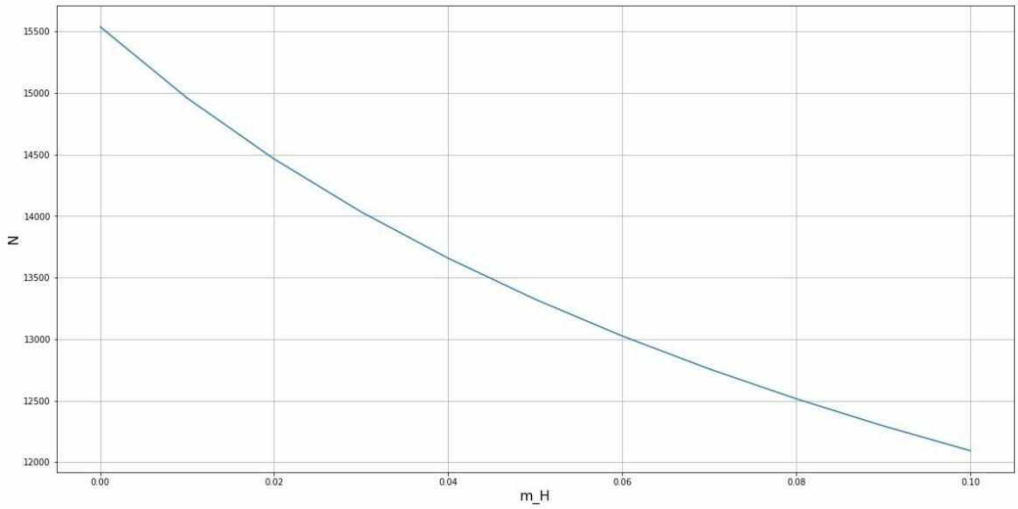


Figure 4. Changing the death rate of hive bees

Sensitivity = 0.1

3.4 Changing the death rate of pupae

Similar to the graph above, the graph here shows that as we increases μ_P from 0.00 to 0,10, N decreases from its original number to below 12000.

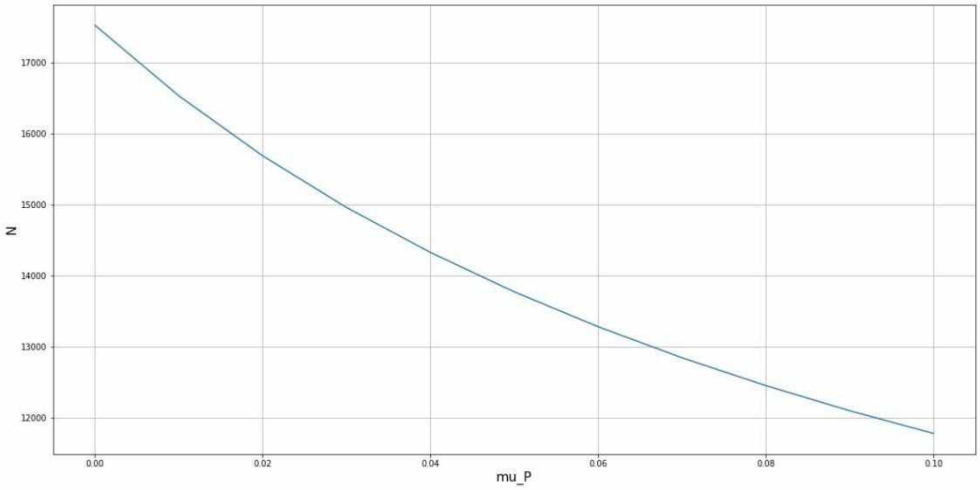


Figure 5. Changing the death rate of pupae

Sensitivity = 0.14

3.5 Changing the death rate of forager bees

Similar to the graph above, the graph here shows that as we increases m_F from 0.00 to 0,10, N decreases from its original number,above 15100, to roughly 14300.

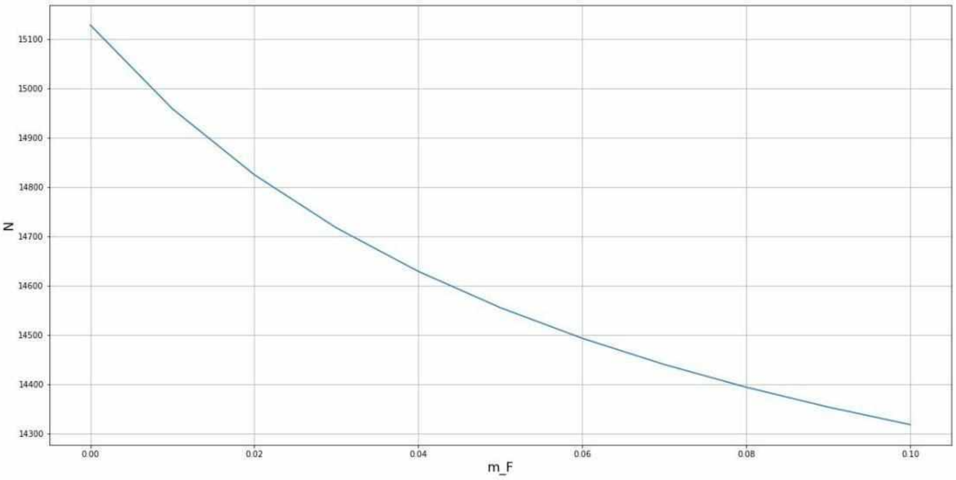


Figure 6. Changing the death rate of forager bees

Sensitivity = 0.02

3.6 Changing the maximum production of eggs

Finally unlike the graphs above, the graph here shows that as we increases $E_{l_{max}}$ from 1000 to 2000, N increases from its original number,below 10000, to above 20000.

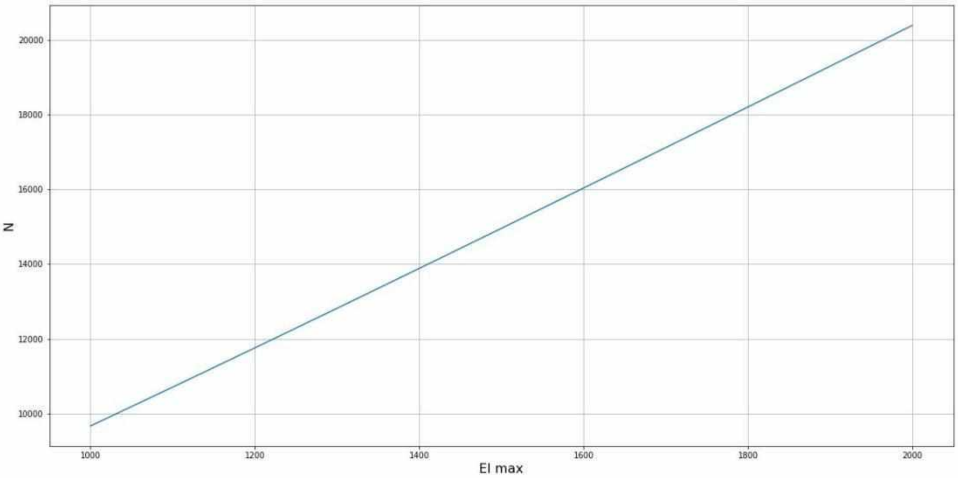


Figure 7. Changing the maximum production of eggs

Sensitivity = 1.05

3.7 Conclusion

From analyzing the results we yielded, for section 4.6 yielded the largest value for sensitivity, it can be concluded that the increase in the maximum production of eggs have the biggest influence on the honeybee population.

4 The Prediction Model

For problem 3, we used cellular automata (CA) to predict the number of hives needed by setting certain rules for the behavior of the honeybees in the hives. CA in general is when a complex output is yielded under the limitation of certain simple patterns. Nevertheless, our solution does not strictly count as using CA, but it is similar in idea since we also used certain patterns to gain the result amount of hives.

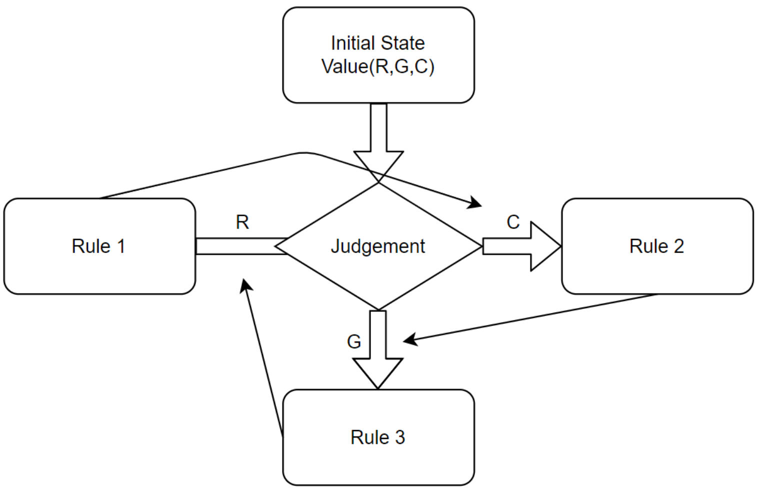


Figure 8. Flow chart

4.1 Assumption and Justification

Assumption 1: Suppose the area to be pollinated is a square, the side length is approximately 285m, divided into 285*285 square grids of 1 m^2, each grid is called "block", and the position of each block is indicated by (x, y).

Justification 1: Since the pollination area required to be calculated as 81,000 square meters, and the square root of 81,000 was equal to approximately 285 meters, the variable length was set to 285 meters. At the same time, dividing the area into squares is conducive to calculating the distance between the lattices, and makes the conclusions more universal.

Assumption 2: The bee's route to the flowers to be collected is approximately a straight line, regardless of height, and is a two-dimensional plane.

Justification 2: First, the effect of altitude on distance is minimal and negligible. Secondly, it is convenient to approximate the flight path of bees as a straight line to calculate the flight distance.

Assumption 3: Suppose that the worker bees collecting honey need to rest for 8 hours a day, and do not rest during the pollination.

Justification 3: According to scientific research, bees need an average of 8 hours of rest a day, and bees work continuously during the process of collecting honey.

Assumption 4: Suppose that all the flowers that bees pass through are successfully pollinated.

Justification 4: The hairy body of a bee is stained with pollen during pollination, and pollination is completed every time it travels from one flower to another, here for the sake of modeling assumptions that all flowers are successfully pollinated.

Assumption 5: Suppose a flower in one square meter requires 10 bees for pollination.

Justification 5: The population model shows that the number of bees in a colony fluctuates between 10,000 and 20,000 with the season, and worker bees account for 99% of a colony. So we can assume that a swarm of bees has 10,000 worker bees and assume that they are all foraging bees. Since a box of bees can cover a thousand square meters, that is, 10,000 bees cover a thousand square meters, resulting in 10 bees covering one square meter.

Assumption 6: Suppose the flight speed of bees is constant, i.e. there is an average speed.

Justification 6: The difference in the flight speed of bees has little effect on the overall pollination results, so the flight speed can be considered constant, taking its average. This assumption can let us quickly and accurately calculate the time required for the bee to fly.

Assumption 7: It is assumed that the pollination period of each plant is fixed.

Justification 7: The fluctuation of the pollination period of plants is small, has little impact on the final solution results, and can be regarded as fixed.

Assumption 8: We assume all of the plants haven't been pollinated yet.

Justification 8： Since the area of plants are going to be pollinated, the plants should all be counted as not been pollinated yet.

Assumption 9: The flowering period is the pollination period in our model.

Justification 9：For most of the plants, their own pollination period and flowering period are very close, including the 7 plants considered in the model. Therefore we view their flowering period as their pollination period.

4.2 Variable Chart

Variable	Description
V_b	Speed of bee when flying
T_{max}	Maximum time for pollinating
y	Initial probability for each point
K	Total number of hives generated randomly
$285 \times 285\boldsymbol{S}$	Matrix of the state of each cell
$285 \times 285\boldsymbol{G}$	Matrix of the probability to the current cell

Table 5. Variable chart for Problem 3

4.3 Model and Explanation

First, we create the matrix $285 \times 285\boldsymbol{S}$ to represent the state of each block, and the matrix $285 \times 285\boldsymbol{G}$ to represent the probability that the block will be pollinated in a round. The matrix is represented by an image as a 285×285 square. Each block represents an area of one square meter. Second, we divide each piece, the cells in the Cellular Automata model, into three states: $cell = 99$ for the hive, $cell = 0$ for flowers that have not been pollinated, and $cells = 1$ for flowers that have completed pollination. Blocks with states 99 and 1 are represented in yellow, while blocks with status 0 are represented in purple. The ultimate goal is to turn all purple into yellow, in other words, to cover the entire area with yellow.

We define the number of hives as K, and the program starts enumerating from $K = 285 \times 285$, that is, when the entire area is covered with hives, there are no flowers to pollinate, so *t* remains in the initial state for 0 seconds, which is naturally less than T_{max}. We define the pollination period as T_{max} (in seconds). For example, the T_{max} (pollination period) of sunflowers is two days or $2 \times 24 \times 60 \times 60 = 172800s$. If t < T_{max} (the cyclic condition for our code), subtract 1 from the number of K and enumerate. Find the corresponding *t* and compare it to T_{max}, and if t < T_{max}, the loop continues.

Following is an example when $K = 200$, which helps us understands the model better.

First, when entering this cycle, 200 hives are randomly generated in a matrix of [285, 285]. Then, the probability of being pollinated is calculated for all unpollinated blocks in the matrix (blocks with a state of 0). We define the calculation model of probability as:

$$y = \frac{1}{(r+1)^4}$$

while r represents the straight-line distance of the block (plant) from the hive:

$$d = (\Delta x^2 + \Delta y^2)^{\frac{1}{2}}$$

The probability of each plant for each hive is different. For example, the probability of bees in hive 1 pollinating this sunflower is different from the probability of bees in hive 2 helping this sunflower to pollinate.

The initial probability yi (y initial) of a single block (unpollinated plant) is the sum of its probabilities for each hive:

$$y_i = y_1 + y_2 + \cdots + y_n$$

The comparative probability yr (y relative) of a single block (plant) is its initial probability divided by the initial probability of all blocks (plants):

$$y_r = \frac{y_i}{y_{inet}}$$

Subsequently, we comprehensively compare the probabilities of all blocks and arrange them from largest to smallest. The higher the probability of the block, the more the state becomes 1 (yellow) first. At each moment, each block with the greatest probability will match the hive closest to it.

The distance d between the block and the hive is defined as:

$$d = (\Delta x^2 + \Delta y^2)^{\frac{1}{2}}$$

, and the time for pollination by bees to the block is defined as:

$$t = \frac{2d}{v}$$

(v is the flight speed of the bees, a fixed value, see assumption and justification) $2d$ takes into account the bees going back and forth between the plant and the hive.

The total time it takes for bees to pollinate is the sum of all t. It is to add together the time t corresponding to each change from a block with a state of 0 to a block with a state of 1 (purple turns yellow).

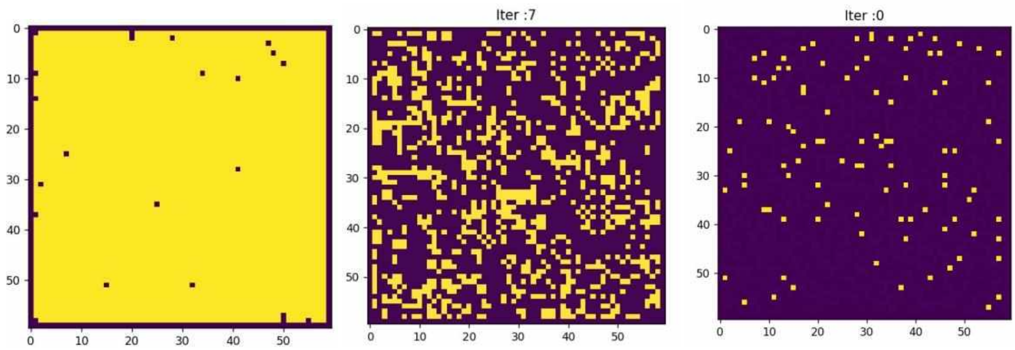


Figure 9. Iteration of pollination model

A more vivid example is that in a 5x5 matrix, there are three hives. After calculating the probability of each unpollinated block, we arrange the blocks. Block A with the highest probability is the closest to the X hive, then

$$t_{AX} = \frac{2d_{AX}}{v}$$

Block B with the second highest probability is the closest to the Y hive (if the distance to X is the largest, block B is skipped, and B is matched to X in the next round), then

$$t_{BY} = \frac{2d_{BY}}{v}$$

The third most probable block C is the closest to Z, then

$$t_{CZ} = \frac{2d_{CZ}}{v}$$

This time for this round,

$$t_{round} = t_{AX} + t_{BY} + t_{CZ}$$

We then add up the t_{round} of each round as

$$t_{net} = t_{r_1} + t_{r_2} + \cdots + t_{r_n}$$

Comparing t_{net} with T_{max}, if t_{net} is smaller than T_{max}, it means that the number of hives K in this cycle will complete pollination in the region within the pollination period. Then subtract the number of hives K by 1 and cycle it again to see if you can complete pollination during the flowering period. The program loops until K reaches or slightly exceeds a critical value, i.e. $t_{net} = T_{max}$ or $t_{net} > T_{max}$. We can then find out at least how many hives the crop needs in the area (81,000 square meters) to complete all pollination during the flowering period.

4.4 Results and Analysis

Name of the Crop	Period of Pollination	How Many Hives Needed
Sunflower	2 Days	7
Apple	3 Days	5
Orange	6 Days	3
Oilseed Rape	10 Days	2
Orchid	80 Days	1

Table 6. Crop Information

It is not difficult to see that the longer the pollination period, the fewer bee colonies are needed.

In real orchards, fruit farmers generally set up a beehive every 4~6 mu (mu is a Chinese unit of area, 1 mu = 666.67 squared meters) of apple land [1] , the 20 mu need about 5~6 beehives, in line with the model concluded that 5 beehives are needed. Citrus has a longer flowering period than common fruits, so the number of beehives required is less than that of common fruits, so the model's conclusion that 3 beehives are needed is also reasonable. Similarly, according to the relevant data, the model's conclusion on the number of beehives required for 20 mu of sunflower, rape, corn, rice, and orchids is also reasonable. [2]

5 Strength and Weakness

5.1 Strength

1. We have conducted a thorough study of bees, including its habits, social division of labor, and life cycle, and considered many factors affecting the number of bee colonies, which makes our model very comprehensive and the conclusions generalized and representative.

2. In our Prediction Model, the Cellular Automata Model can vividly demonstrate the process of bee pollination. The final results are in line with reality, further demonstrating the accuracy and reliability of our conclusions.

3. Our prediction model for question 3 has a very visualized result. We can clearly see how the bees pollinate under our assumptions over time.

5.2 Weakness

1. Not too complete. Still have flaws on the regulation factor.
2. Some assumptions are not very close to daily life due to some of our proved not-reliable sources.
3. Some results not very easy to understand directly from the graphs. (Q2)
4. Differential equations are sensitive to the change in parameters.
5. In this report, we only considered one species of honeybees where no competitions between different species took place.

6 References

Bagheri, S., & Mirzaie, M. (2019). A mathematical model of honey bee colony dynamics to predict the effect of pollen on colony failure. PLoS ONE, 14.

Chen, J., DeGrandi-Hoffman, G., Ratti, V., & Kang, Y. (2021). Review on mathematical modeling of honeybee population dynamics. Mathematical biosciences and engineering : MBE, 18 6, 9606-9650 .

Khoury, D.S., Myerscough, M.R., & Barron, A.B. (2011). A Quantitative Model of Honey Bee Colony Population Dynamics. PLoS ONE, 6.

Chen, J., Messan, K.S., Messan, M.R., DeGrandi-Hoffman, G., Bai, D., & Kang, Y. (2020). How to model honeybee population dynamics: stage structure and seasonality. Mathematics in Applied Sciences and Engineering.

Lv Xiaowu, Lv Yumin. Biological knowledge of bees pollinating apple flower[J]. China Bee Industry, 2006, 57(7):1.

Liu Kuanding. Pollination measures of bees in orchards[J]. Shanxi Agriculture, 2005(9):2.

(2013). Factors Affecting Global Bee Health Honey Bee Health and Population Losses in Managed Bee Colonies.

Berto, F., & Tagliabue, J. (2017, August 22). Cellular automata. Retrieved November 16, 2022, from https://plato.stanford.edu/entries/cellular-automata/

02: Life cycle - serendipi-bee. (2019, July 17). Retrieved November 16, 2022, from https://www.serendipi-bee.ca/basics/intro/life-cycle/

Honeybee. (n.d.). Retrieved November 16, 2022, from https://www.britannica.com/animal/honeybee

Revive a Bee. (2021, January 31). 3 types of bees in a hive - the mystery of the hives inhabitants revive a bee. Retrieved November 16, 2022, from https://reviveabee.com/3-types-of-bees-in-a-hive/

7 Appendix

For Problem 1 – The Population Model:

Parameter	Description	Estimated Value
λ_E	Rate of eggs converting into larvae	$1/3(/day)$
λ_L	Rate of larvae converting into pupae	$1/5(/day)$
λ_P	Rate of pupae converting into hive bees	$1/12(day)$
f_g	Mass of food gathered per forager bee	$0.11(/day)$
f_{c_L}	Mass of food consumed per larva	$0.036(/day)$
f_{c_H}	Mass of food consumed per hive bee	$0.014(/day)$
f_{c_F}	Mass of food consumed per forager bee	$0.014(/day)$
t_{start}	Time started	0

t_{flower}	Time gap between season and florescence	$45(day)$
T_{summer}	Average temperature in summer	$34(°C)$
T_{winter}	Average temperature in winter	$10(°C)$
φ_{max}	Maximum relative concentration of food	1.75
φ_{min}	Minimum relative concentration of food	0.25
T_{b_1}	Best temperature in bell curve function 1	$27(°C)$
T_{b_2}	Best temperature in bell curve function 2	$17(°C)$
T_{b_3}	Best temperature in bell curve function 3	$27(°C)$
T_{b_4}	Best temperature in bell curve function 4	$27(°C)$
T_{b_5}	Best temperature in bell curve function 5	$27(°C)$
C_1	Constant in bell curve function 1	0.003
C_2	Constant in bell curve function 2	0.001
C_3	Constant in bell curve function 3	0.003
C_4	Constant in bell curve function 4	0.003
C_5	Constant in bell curve function 5	0.002
$E_{l_{max}}$	Maximum egg laying rate	1500
α_N	Regulation constant related to N	5000
α_H	Regulation constant related to H	1000
β	Regulation constant related to f for E_l	$100(g)$
β_L	Regulation constant related to f for S_L	$500(g)$
β_H	Regulation constant related to f for S_H	$200(g)$
β_F	Regulation constant related to f for S_F	$200(g)$
β_R	Regulation constant related to f for R	$700(g)$
γ_E	Self-regulation constant related to E	0.2
γ_L	Self-regulation constant related to L	0.2
γ_P	Self-regulation constant related to P	0.2
σ_{E_l}	Fundamental regulation constant for E_l	0.9
σ_E	Fundamental regulation constant for S_E	0.9
σ_L	Fundamental regulation constant for S_L	0.9
σ_P	Fundamental regulation constant for S_P	0.9
σ_H	Fundamental regulation constant for S_H	0.9
σ_F	Fundamental regulation constant for S_F	0.9
μ_E	Death rate of E due to parasites, pesticides, etc.	0.03
μ_L	Death rate of L due to parasites, pesticides, etc.	0.03
μ_P	Death rate of P due to parasites, pesticides, etc.	0.03
μ_H	Death rate of H due to parasites, pesticides, etc.	0.02

μ_F	Death rate of F due to parasites, pesticides, etc.	0.02
m_H	Natural death rate of H	0.02
m_F	Natural death rate of F	0.02
δ	Recruitment coefficient of hive bees	$1/15$

Table 3. Parameters Description for Problem 1

Variable initial condition
$E(0) = 5000$
$L(0) = 5000$
$P(0) = 5000$
$H(0) = 3000$
$F(0) = 3000$
$f(0) = 1600(g)$

Table 4. Variable initial condition for Problem 1

For Problem 3 - Code Segments:

```
import matplotlib.pyplot as plt
import numpy as np

time=200
time_max=86400*6 #second
"""
一天= 86400 秒
工作时间=86400秒-28800秒=57600秒

向日葵   2天              油菜 10天
柑橘 6天    苹果  3天
玉米7天   兰花 80天
水稻 2h
"""
bee_amount=10000/10
x=0
class Q3(object):

  def __init__(self, cells_shape):
    """
    Parameters
    ----------
    cells_shape :A tuple that represents the size of the canvas.

    Examples
    --------
    game = Q3((20, 30))

    """

    self.cells = np.zeros(cells_shape)

    real_width = cells_shape[0] - 2
    real_height = cells_shape[1] - 2

    self.cells=np.zeros(cells_shape)
    self.row=np.zeros((81225,3))

    for j in range(1,time):
          xc=np.random.randint(1,285)
          yc=np.random.randint(1,285)
          if self.cells[xc,yc]==0:
            self.cells[xc,yc]=99
            self.row[j,1]=xc
            self.row[j,2]=yc
          else:
            j-=1
    self.timer = 0
    self.mask = np.ones(9)
    self.mask[4] = 0
    self.gailu=np.zeros(cells_shape)
```

```
def update_state(self):
    """更新一次状态"""
#
    buf = np.zeros(self.cells.shape)
    cells = self.cells
    gailu = self.gailu
    summ=0
    for l in range(1,285):
        for i in range(1, cells.shape[0] - 1):
            for j in range(1, cells.shape[0] - 1):
                if(self.cells[i,j]==99):
                    continue
                if 1-np.sqrt((i-self.row[l,1])**2+(j-self.row[l,1])**2)==0:
                    continue
                x=1/((1-np.sqrt((i-self.row[l,1])**2+(j-self.row[l,1])**2))**4)
                gailu[i,j]+=x
                summ+=x
    find=np.random.uniform(0,summ)
    noww=0
    f=0
    g=0
    while 1:
        for i in range(1, cells.shape[0] - 1):
            for j in range(1, cells.shape[0] - 1):
                if self.cells[i,j]==99:
                    continue
                noww+=gailu[i,j]
                if noww>=find:
                    f=1
                    if self.cells[i,j]!=1:
                        self.cells[i,j]=1
                        g+=1
                        break
            if f==1:
                break
        if g==bee_amount:
            break

def plot_state(self):
    """画出当前的状态"""
    print(self)
    #plt.title('Iter :{}'.format(self.timer))
    #plt.imshow(self.cells)
    #plt.show()
```

```
  def update_and_plot(self, n_iter):
    """更新状态并画图
    Parameters
    ----------
    n_iter : 更新的轮数
    """
    plt.ion()
    f=0
    while(f==0):

      time_count=0
      plt.title('Iter :{}'.format(self.timer))
      plt.imshow(self.cells,cmap="YlGn_r")
      self.update_state()
      f=1
      for i in range(1, self.cells.shape[0] - 1):
        for j in range(1, self.cells.shape[0] - 1):
            if self.cells[i,j]==0:
                f=0
      time_count+=0.278*x*2
      plt.pause(2)
    if time_count>time_max:
      print(time)
      SystemExit
    plt.ioff()

if __name__ == '__main__':
  while time>0:
    game = Q3(cells_shape=(285,285))
    game.update_and_plot((200))
    time-=1
```